Journal of Mechanical Science and Technology 24 (11) (2010) 2175~2180
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-010-0904-4

Design of a vibrating MEMS gyroscope considering design variable uncertainties[†]

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(Manuscript Received January 22, 2010; Revised April 22, 2010; Accepted August 1, 2010)

Abstract

Recently, several micro electro-mechanical systems (MEMS) such as a MEMS gyroscope have been developed by using micro manufacturing technologies. Micro scale products, however, usually have a relatively large manufacturing uncertainty compared to normal macro scale products. It is quite expensive to lower the variance of material properties as well as the geometric properties of a micro scale product. The material and geometric uncertainties caused by a micro manufacturing process inevitably lead to the uncertainty of the product performance. Therefore, to achieve a reliable design of a product, the performance uncertainty of the product, which is often expressed by the variance or the standard deviation, needs to be estimated in a reliable way. In this paper, the equations of motion of a MEMS gyroscope model are derived to analyze the system performance indices (sensitivity and bandwidth). The mean values of the design variables are determined from the requirements of product size, maximum vibration amplitude, and driving frequency. Then the standard deviation of some critical design variables is determined from the performance requirements. Finally, a statistical analysis procedure based on sample statistics is proposed to estimate the confidence interval of the performance index statistics.

Keywords: MEMS gyroscope; Sensitivity; Bandwidth; Uncertainty; Statistical analysis

1. Introduction

A gyroscope is an angular rate sensor which has been used for several decades to measure the input angular velocity of an inertial system. Conventional gyroscopes are relatively large and expensive so that they are not proper for general industrial and household applications. To overcome such limitations, a micro scale vibrating type gyroscope [1] in which the angular rate could be obtained by measuring the Coriolis acceleration was developed recently. In the early 1990's, micromachining technology advancement enabled the development of several types of miniaturized low-cost MEMS gyroscopes having much improved performances [2-5].

Design methods for mechanical systems can be classified into two groups: deterministic methods and stochastic methods. When a deterministic design method is employed, fixed values of design variables which satisfy the performance requirements of the design problem need to be found. In most practical situations, however, the values of design variables cannot be fixed due to the uncertainties caused by manufacturing error, wear, creep and environmental change. Micro scale manufacturing especially results in relatively large error. Therefore, reliable product quality control cannot be guaranteed with a deterministic design approach. The product performance uncertainty caused by several design variable uncertainties can be only reliably estimated by using a statistical design approach. Therefore, a stochastic method should be employed to consider the uncertainty.

In a stochastic method, the statistics of design variables are employed to estimate the performance uncertainties of a system. Hartenberg and Denavit [6] addressed the issue of motion uncertainty in linkages due to tolerances. Garrett and Hall [7] developed a method to determine the statistics of mechanical errors due to linkage length tolerances and joint clearances. To analyze the motion uncertainty caused by linkage length tolerances and joint clearances more efficiently, Lee and Gilmore [8] proposed a statistical formulation in which the first-order Taylor series expansion was employed. More recently, Kim et al [9] introduced an analytic procedure to estimate the modal characteristic uncertainties of general multi-body systems based on the first order Taylor series expansion method.

In the previous studies for stochastic design methods mentioned so far, the statistics of design variable populations is assumed to be known. To obtain the statistics of design variable populations, complete enumeration has to be made. However, for most of engineering problems, it is extremely expensive (if not impossible) to carry out such a complete enumeration. Only limited samples of design variables can be practically obtained. The statistics of the populations of design variables and performance need to be estimated based on the

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[†] This paper was recommended for publication in revised form by Associate Editor Keum-Shik Hong

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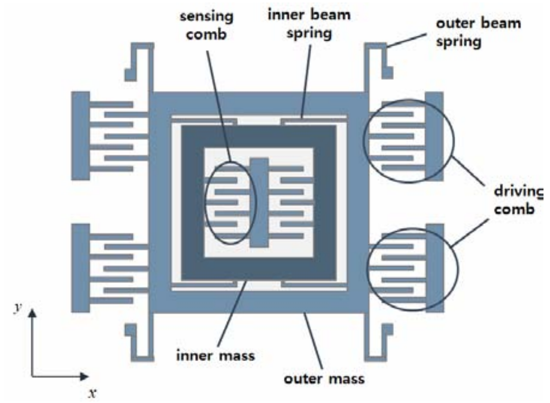


Fig. 1. Actual structure of a vibrating MEMS gyroscope.

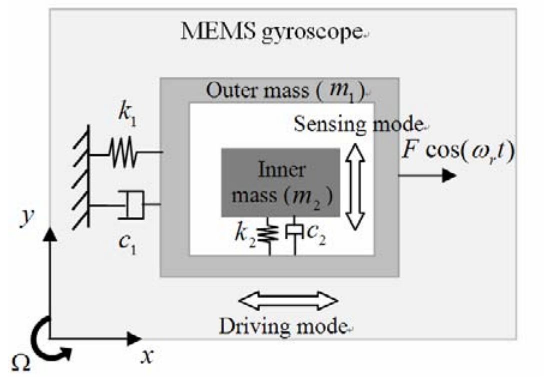


Fig. 2. Idealized schematic diagram of the MEMS gyroscope.

sample statistics with some confidence level. Recently, Kim and Yoo [10] proposed a method to estimate the modal characteristic uncertainties of a structural system based on design variable samples and applied the method to an AFM system which is idealized as a single degree of freedom model.

In this study, a vibrating MEMS gyroscope is designed by considering its design variable uncertainties. The MEMS gyroscope is idealized by a two-degrees-of-freedom model. The design procedure of the system is given as follows. Initial values of the design variables of the MEMS gyroscope are determined from the requirements of size, maximum vibration amplitude, and driving frequency. Then the effects of design variables on the natural frequency difference (which influences performance indices of the MEMS gyroscope) are investigated through parameter study. Then the statistics of design variables which meet the performance requirements are obtained. Finally, with a confidence level, the confidence intervals of performance indices are estimated through the statistical approach proposed in this study.

2. MEMS gyroscope

2.1 Structure and measuring principle of MEMS gyroscope

In the present study, the effects of design variable uncertainties on the performance uncertainties are evaluated based on design variable sample statistics in order to design a MEMS gyroscope. The structure of a vibrating MEMS gyroscope is shown in Fig. 1. This MEMS gyroscope has a symmetric structure with two masses and it vibrates along the x-axis (driving direction) while the Coriolis acceleration is measured along the y-axis (sensing direction). The outer mass is supported by outer beam springs attached to the substrate and it undergoes a prescribed vibration along the x-axis (driving direction).

The vibrating motion of the outer mass is prescribed by an electro static harmonic excitation force. On the other hand, the inner mass is supported by inner beam springs attached to the outer mass. Relative to the outer mass, the inner mass can move along the y-axis (sensing directions). If the outer mass is excited by an electro static harmonic force induced by the driving combs, two masses oscillate along the driving direction. To obtain a large vibration amplitude of the inner mass, the driving frequency should be close to the natural frequency (f_1) of the system. If the MEMS gyroscope rotates around the z-axis (normal to the driving and sensing directions) with an angular rate of Ω, the inner mass will undergo Coriolis acceleration and vibrate along the sensing direction (y-axis). The output vibration amplitude caused by the Coriolis acceleration is nearly proportional to the input angular rate. Therefore, if the vibration amplitude of the inner mass is measured, the angular rate of the rotational motion can be obtained.

2.2 Derivation of the equations of motion

To analyze the performances of the vibrating MEMS gyroscope, the equations of motion of the system need to be derived. Fig. 2 shows a simplified model of the MEMS gyroscope which is idealized to have two degrees of freedom. As shown, the simplified two-degrees-of-freedom model oscillates along the driving direction (x-axis) and the sensing direction (y-axis). The simplified model consists of the outer mass m_1, the inner mass m_2, the outer beam spring k_1, the inner beam spring k_2, the driving direction damping coefficient c_1 and the sensing direction damping coefficient c_2. The excitation force along the driving direction is given as $F\cos\omega t$. If the MEMS gyroscope does not rotate, m_1 and m_2 oscillate only along the driving direction (x-axis). However, if the MEMS gyroscope rotates around the z-axis with an angular rate Ω, m_2 will oscillate along the sensing direction due to the Coriolis acceleration effect. Therefore the inner mass vibrates along the y-axis with amplitude proportional to the angular rate Ω.

The equations of motion of the MEMS gyroscope can be derived as follows:

$$(m_1 + m_2)\ddot{x} + c_1\dot{x} + k_1 x - 2m_2\Omega\,\dot{y} = F\cos\omega t\,, \tag{1}$$

$$m_2\ddot{y} + c_2\dot{y} + k_2 y + 2m_2\Omega\,\dot{x} = 0\,. \tag{2}$$

From the equations, one can easily see that the inner mass does not oscillate relative to the outer mass when the angular rate Ω is zero. If the angular rate is given with a non-zero

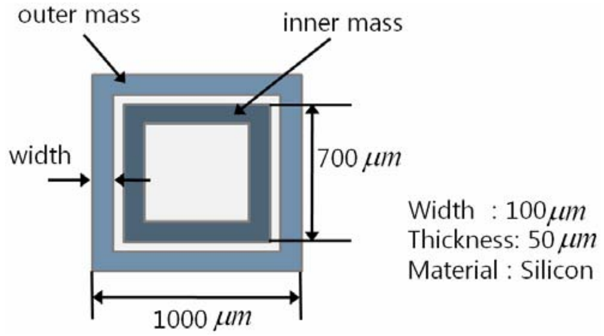


Fig. 3. Geometry and material of the MEMS gyroscope.

value, the oscillating motion of the inner mass will be coupled with that of the outer mass. So once the vibration amplitude of the inner mass is measured, the angular rate Ω can be obtained by using the relationship between the angular rate and the vibration amplitude of the inner mass in the sensing direction.

3. Design of the MEMS gyroscope

3.1 Basic design step

In the basic design step, one should determine the design variables which satisfy some basic design requirements related to system size and maximum amplitude. Design variables consist of six parameters: mass (m_1, m_2), spring constant (k_1, k_2) and damping coefficient (c_1, c_2) appearing in the equations of motion. Mass parameters are determined by the material density and the size of the MEMS gyroscope shown in Fig. 3. Typically, the MEMS gyroscope is made of silicon which has the density of 2330 kg/m^3. Since the outer and inner masses have square shapes, m_1 and m_2 can be determined as 41.94 μg and 27.96 μg, respectively. The spring constant and damping coefficient are related to the vibration amplitude which should be constrained by the MEMS gyroscope size. The maximum amplitudes of the inner mass and the outer mass should not exceed 2 μm while the natural frequency of the system is 7500 Hz which is given from the system driving frequency. Then the spring constants k_1 and k_2 can be obtained as 155.23 N/m and 62.09 N/m (using the natural frequency) and the damping coefficients c_1 and c_2 are respectively chosen as 1.098 x 10^{-6} kg/s and 4.391 x 10^{-6} kg/s which satisfy the maximum amplitude requirement. Design variables obtained from the basic design step are arranged in Table 1. These values are obtained simply by considering the size and motion constraint requirements. The design variables should be modified to satisfy some performance requirements of the MEMS gyroscope.

3.2 Performance design step

An important performance index of a MEMS gyroscope is the sensitivity which is defined as the sensing direction vibration amplitude per the angular rate change. For a MEMS gyroscope which can precisely measure the input angular rate, it is better to increase the sensitivity. The sensitivity S of a MEMS gyroscope can be obtained as follows:

Table 1. Design variables determined from the basic design step.

Design variables	Data
m_1	41.94 μg
m_2	27.96 μg
k_1	155.23 N/m
k_2	62.09 N/m
c_1	1.098 x 10^{-6} kg/s
c_2	4.391 x 10^{-6} kg/s

$$S = \frac{dY}{d\Omega}. \tag{3}$$

where Ω is the input angular rate and Y is the vibration amplitude of the inner mass for the angular rate.

Another important performance index of a MEMS gyroscope is the bandwidth which is defined as the angular frequency range of the input angular rate which satisfies the following inequality constraint equation.

$$20\log\left(\frac{S_r}{S_0}\right) > -3. \tag{4}$$

where S_0 is the sensitivity when input angular rate is given constant and S_r is the sensitivity when the input angular rate is given as a harmonic function with angular frequency ω. So the maximum value of ω which satisfies Eq. (4) is the bandwidth of the MEMS gyroscope.

Two performance indices mentioned above are related to the difference between two natural frequencies: that of the driving direction mode and that of the sensing direction mode. The sensitivity of the MEMS gyroscope increases when the difference of the two natural frequencies decreases. On the other hand, the bandwidth increases when the difference of the two natural frequencies increases. Therefore, the difference of the two natural frequencies has to be decided within a certain range of difference. In this study, the MEMS gyroscope is designed with the following requirements. The sensitivity should not be less than 2 nm/rad/sec and the bandwidth should not be less than 50 Hz. Table 2 summarizes the two design requirements. Relations between the performance indices and the difference of two natural frequencies are obtained as shown in Fig. 4. From the results shown in Fig. 4(a) and 4(b), the difference of two natural frequencies should be less than 151.22 Hz and not less than 25.60 Hz. That is, to satisfy the two design requirements, the difference of two natural frequencies should be between 25.60 Hz and 151.22 Hz.

Table 3 shows the sensitivity and the bandwidth when the difference of two natural frequencies has the lower limit value (25.60 Hz), the upper limit value (151.22 Hz) and an intermediate value (88.41 Hz), respectively.

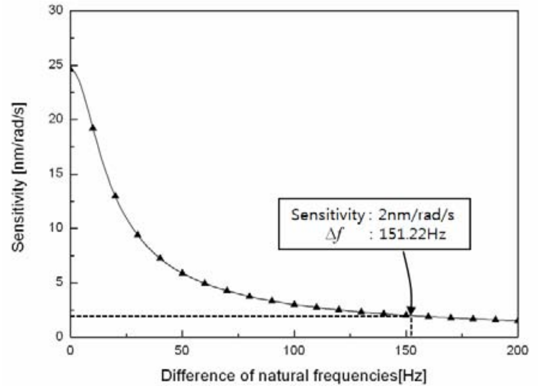
Now, the sensitivity and the bandwidth at the intermediate

Table 2. Design requirements of the MEMS gyroscope.

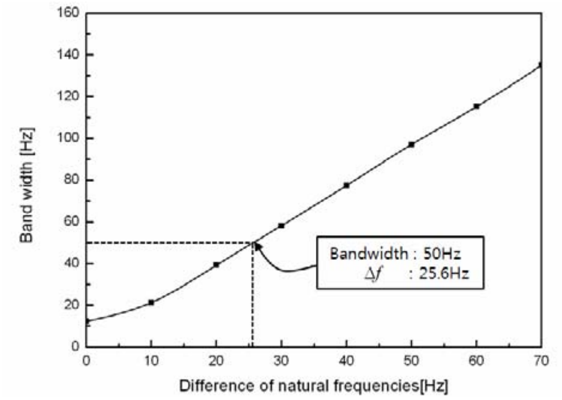
Performance index	Minimum design requirement
Sensitivity	2 nm/rad/s
Bandwidth	50 Hz

Table 3. Sensitivity and bandwidth at each Δf.

Performance index	Δf=25.60 Hz	Δf=88.41 Hz	Δf=151.22 Hz
Sensitivity	10.93 nm/rad/s	3.20 nm/rad/s	2 nm/rad/s
Bandwidth	50 Hz	168.1 Hz	286.7 Hz



(a) Sensitivity versus the difference of natural frequencies



(b) Bandwidth versus the difference of natural frequencies

Fig. 4. Performance indices vs. the difference of natural frequencies.

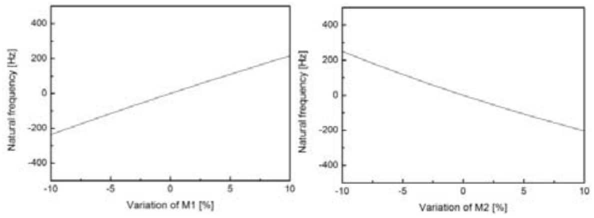
value of the natural frequency difference are chosen as the performances of the MEMS gyroscope. The corresponding values of design variables are shown in Table 4. To investigate the effects of design variables on the variation of the natural frequency difference of the MEMS gyroscope, a parameter study is carried out. In the previous section, it was shown that the natural frequency difference could influence the sensitivity and the bandwidth significantly. Therefore, the variations of the natural frequency difference versus the design variable variations of 20% (-10% to 10%) of the design variable mean values are shown in Fig. 5. From the results of the parametric study, it can be shown that the design variables m_1, m_2, k_1 and k_2 influence the natural frequency difference of the MEMS gyroscope significantly while the design variables c_1 and c_2 influence little.

Table 4. Mean values of design variables.

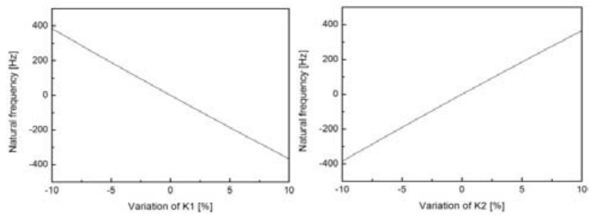
Design variable	Determined mean value
m_1	41.94 μg
m_2	27.96 μg
k_1	155.23 N/m
k_2	63.65 N/m
c_1	1.098×10^{-6} kg/s
c_2	4.391×10^{-6} kg/s

Table 5. Statistics of the important design variables.

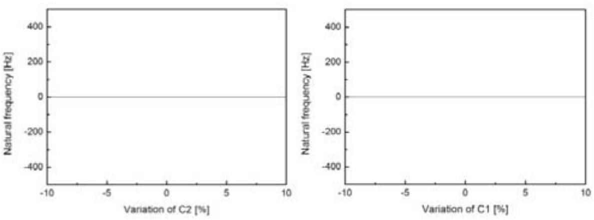
Design variables	Mean	STDEV
m_1	41.94 μg	0.067 μg
m_2	27.96 μg	0.045 μg
k_1	155.23 N/m	0.25 N/m
k_2	63.65 N/m	0.10 N/m



(a) m_1 and m_2



(b) k_1 and k_2



(c) c_1 and c_2

Fig. 5. Effects of design variables on the variation of the natural frequency difference.

The design variables m_1, m_2, k_1 and k_2 are assumed to have normal distribution in this study. Since the range of the natural frequency difference is obtained from Fig. 4, the standard deviations of the design variables can be determined using the sensitivities of the natural frequency difference at the four design variable mean values. The statistics of the design vari-

Table 6. Estimated statistics of performance indices.

Performance index	Mean	STDEV
Sensitivity	3.2	0.2
Bandwidth	168.1	19.6

ables with six-sigma level are shown in Table 5.

If the statistics of the design variables are determined as shown in Table 5, the statistics of the sensitivity and the bandwidth can be obtained, too. The statistics of the performance indices are shown in Table 6.

4. Performance estimation of the MEMS gyroscope

4.1 Statistical estimation based on variable sample statistics

When the distributions of design variable populations are known, the distribution of a system performance measure can be obtained as follows [9]:

$$Y = g(b_1, b_2, \ldots, b_i), \tag{5}$$

$$E(Y) = g(\mu_{b_1}, \mu_{b_2}, \ldots, \mu_{b_i}), \tag{6}$$

$$Var(Y) = \sum_{j=1}^{i} \left(\frac{\partial g}{\partial b_j} \right)^2 Var(b_j) \tag{7}$$

where Y is the performance measure, b_i's are design variables, μ_{b_i} is the mean of b_i, $Var(b_i)$ is the variance of b_i, $E(Y)$ is the mean of Y, and $Var(Y)$ is the variance of Y. In Eqs. (6) and (7), if the means and the variances of the design variables are known, the mean and the variance of a performance measure Y can be obtained. If the population statistics of the design variables are not known, however, they should be estimated before the system performance statistics are estimated. The first step is the population statistics estimation of each design variable. In this step point estimation is employed and point estimators of design variables are obtained respectively. Using Eq. (6) and Eq. (7), pseudo statistics of system performance are obtained. Then applying the confidence interval estimation method, the population statistics of system performance can be estimated. The confidence interval estimation procedure mentioned so far is shown in Fig. 6.

4.2 Confidence interval estimation of performance statistics

Using the procedure mentioned in Section 4.1, the confidence intervals of the statistics of two performance indices are estimated. For the estimation, a sample size of 100 was employed and the confidence level of 99% is required. The estimation results are obtained and arranged in Table 7. To verify the results, Monte Carlo simulation is conducted and the results are given in Fig. 7. As shown, the reliabilities are converging to 99% as the number of times for random sample choice increases.

Table 7. Confidence intervals of two sets of performance statistics.

Performance index	Mean		Standard deviation	
	Lower	Upper	Lower	Upper
Sensitivity	3.16	3.23	0.18	0.23
Bandwidth	166.5	169.8	15.5	20.7

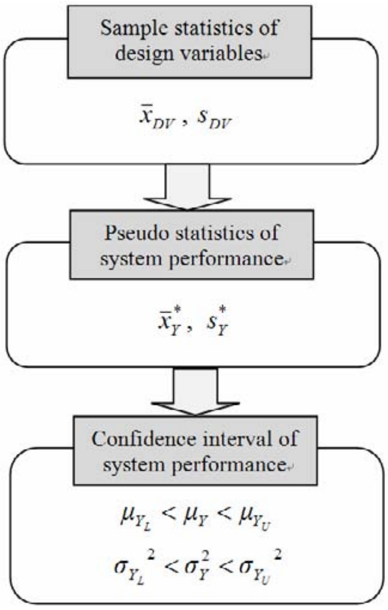


Fig. 6. Procedure of estimating performance confidence interval.

5. Conclusion

A design procedure for a MEMS gyroscope is proposed based on a statistical analysis method. Initial values of design variables are determined from the size and the vibration amplitude requirements. Two performance indices (sensitivity and bandwidth) which can be calculated from the equations of motion are defined. To satisfy the performance requirements, a parameter study is carried out first and the statistics of influential design variables are determined by employing 6 sigma level. Lastly, the confidence intervals of the performance indices are estimated based on sample statistics of design variables. The procedure proposed in this study can be employed for robust design of the MEMS gyroscope.

Acknowledgment

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (**NRF**) funded by the Ministry of Education, Science and Technology (No. 2010-0016395).

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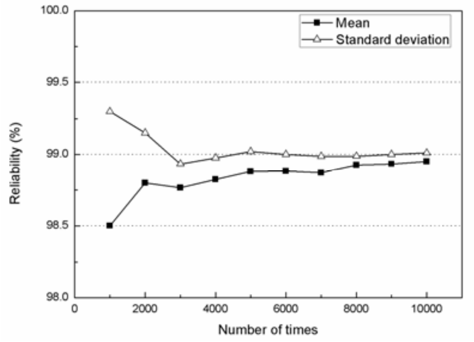
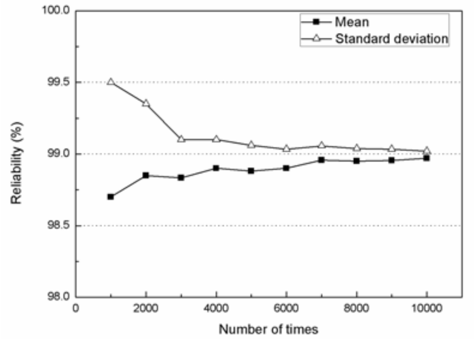


(a) Sensitivity



(b) Bandwidth

Fig. 7. Verification of confidence interval estimation.

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